

Hippocampus

Space-Filling Bearings in Three Dimensions

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We present the first space-filling bearing in three dimensions. It is shown that a packing which contains only loops of an even number of spheres can be constructed in a self-similar way and that it can act as a three-dimensional bearing in which spheres can rotate without slip and with negligible torsion friction.

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Space-filling bearings have been introduced in several contexts, such as in explaining the so-called seismic gaps [1,2] of geological faults in which two tectonic plates slide against each other with a friction much less than the expected one, without production of earthquakes or of any significant heat. Space-filling bearings have also been used as toy models for turbulence and can also be used in mechanical devices [3]. Two-dimensional space-filling bearings have been shown to exist and a discrete infinity of realizations has been constructed [4,5]. The remaining question still open is: Do they also exist in three dimensions? This question is of fundamental importance to the physical applications.

In this Letter, we will report the discovery of a self-similar space-filling bearing in which an arbitrary chosen sphere can rotate around any axis and all the other spheres rotate accordingly without any sliding and with negligible torsion friction.

In two dimensions, different classes of space-filling bearings of disks have been constructed in Refs. [4,5] by requiring the loops to have an even number of disks, since in two dimensions this is obviously a necessary and sufficient condition for disks to be able to rotate without any slip. Successive disks must rotate, in alternation, clockwise and counterclockwise in order to avoid frustration.

The situation in three dimensions is different from two dimensions in two ways: The axes of rotation need not be parallel, and the centers of spheres in a loop may not all lie in the same plane. As a result, even in an isolated odd loop spheres could rotate without friction. But, as we will see, in the packings with an infinite number of interconnecting loops, we could construct unfrustrated configurations of rotating spheres when all loops have an even number of spheres. Such a packing is *bichromatic*; i.e., one can color the spheres using only two different colors in such a way that no spheres of the same color touch each other, as shown in Fig. 1.

No three-dimensional space-filling bearing has been known until now. The classical Apollonian packing is

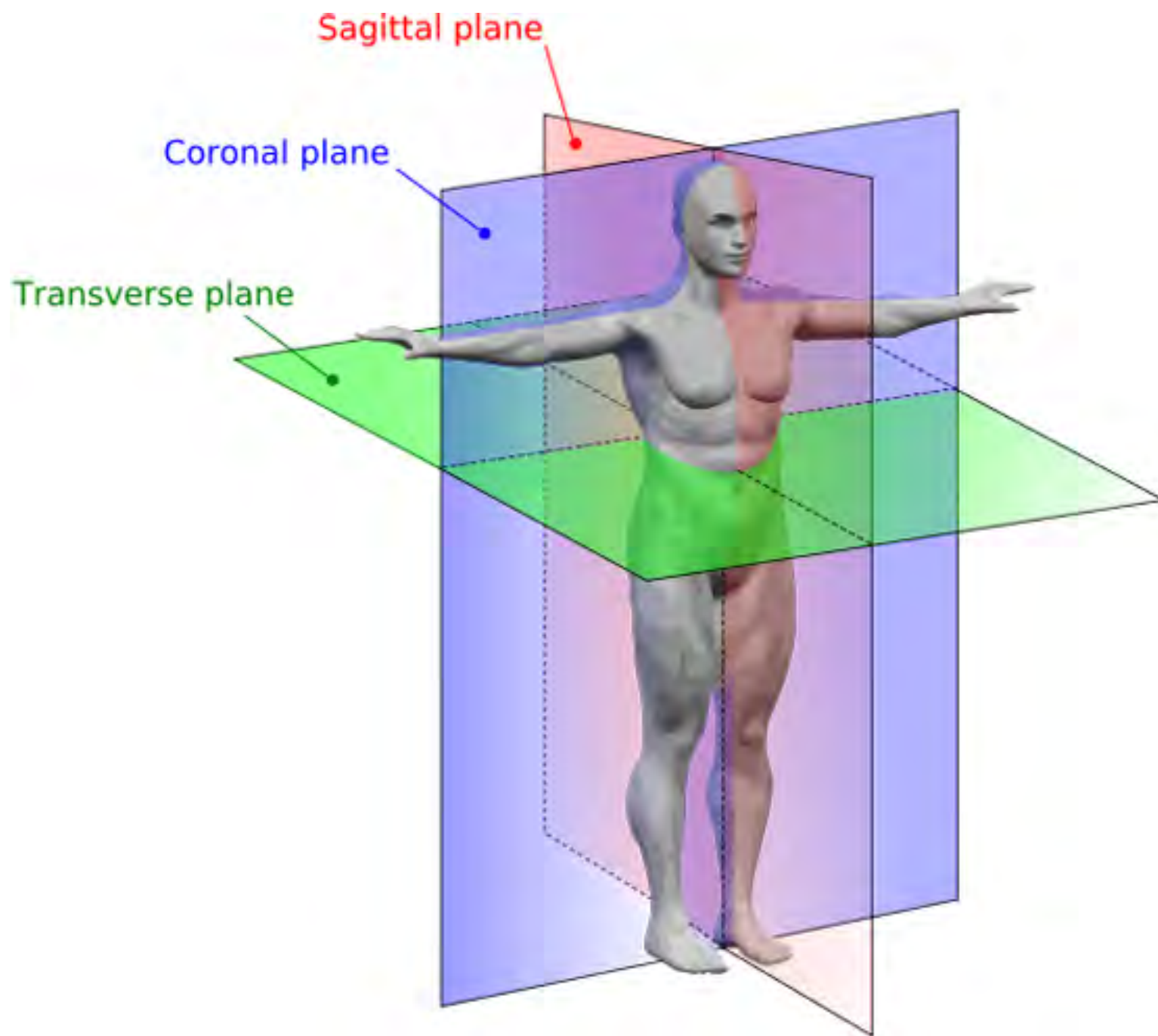
space filling and self-similar but not a bearing since at least five colors are needed to assure different colors at each contact. This packing can be constructed in different ways [6,7]. By generalizing the inversion technique used in Ref. [6] to other *Platonic solids* than the tetrahedron (the base of 3D Apollonian packing), we were able to construct five new packings including a bichromatic one. Details on the construction algorithm and on the complete set of new configurations will be published elsewhere [8]. We give here only a qualitative description of this technique for the bichromatic packing.

Let us consider filling a sphere of unit radius. The filling procedure is initialized by placing seven initial spheres on the vertices and the center of a regular octahedron inside the unit sphere, so that the spheres on the vertices do not touch each other but touch the unit sphere and the one in the center. Further spheres are inserted by inversion [9] such that this topology can be preserved on all scales, imposing that all spheres are on vertices or centers of (deformed) octahedra.

The inversions are made iteratively with respect to nine *inversion spheres* [10]: One inversion sphere is concentric with the unit sphere, and is perpendicular to the six initial spheres on the vertices of the octahedron [11]. Inversion



FIG. 1. Bichromatic packing of spheres with octahedral symmetry. No two spheres of the same color touch each other. The image on the left shows the initial configuration and the first generation of inserted spheres.









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My Autobiography
William C. C. Chen

C C Chen' I am a disciple of Professor Cheng Man-chang, grand master of Yang's style T'ai Chi Ch'uan. His knowledge of poetry, Chinese medicine, and T'ai Chi Ch'uan. I learned poetry in collage at the age of 18 and taught at school. People addressed him as Professor Cheng. It was an honor to be named as my T'ai Chi Ch'uan master.

A Regal Bearing

When physicist Hans J. Herrmann of the University of Stuttgart in Germany heard a 1985 talk about tectonic plates sliding past each other with unexpectedly low friction, he began mulling over the nature of space-filling groups of ball bearings. He soon found theoretical arrays of two-dimensional disks that all turn in harmony, but a three-dimensional version proved elusive—no matter the arrangement, some balls would slip and rub, instead of turning against their neighbors. The physicist and his colleagues have now solved the problem theoretically.

Imagine a sphere with six smaller spheres placed inside like the corners of a regular octahedron. The remaining space inside the big sphere can be completely filled with ever smaller spheres in a fractal pattern by a mathematical technique called inversion. Turn one sphere, and the rest turn without rubbing. A real bearing based on this model must consist of finitely many spheres, which Herrmann says would still be frictionless unless the balls were somehow forced out of place. Turn to the January 30 *Physical Review Letters* for the head-spinning result.

—JR Minkel



ROLL ON: This theoretical model has ball bearings arranged so that none slips against another.



Interlocking Hearts from Merged Trinity Knots



**This is an concept piece created by merging left hand and right hand trefoil knots
The arms contract as they come together and expand as they separate**

**The trefoil knot is the most basic of knots and comes in two shapes which are
mirror images of each other: the Right hand knot and the Left hand knot**

**I became facinated with trefoil knots 8 years ago while I was developing
routines for the movement system I had created. I wanted to go from the circles
I made on the horizontal plane to circles on the other two planes. After
experimenting I realized the most efficient and graceful way to go between
the planes is by following the path of the Trefol Knot - I was hooked. I made
many models out of copper tubing and plastic but it wasn't until I came across
bronze that I could express the variation and expansion of the knots**

**Since symmetrical synchronized movements are at the foundation of my system
(Octayoga), it did not take me long before I was joining the left and right hand
knots together. Simultaneously following (tracing) the paths of the joined knots
with my hands produced a special feeling of deep regenerative healing unlike
anything I had experienced before. With further reflection, I realized that the
movements made when tracing the joined left and right knots were the natural,
universal human gestures that represent wonder, awe, and reverence**

**The fused knots look like a pelvis or spine. I suspect that vertebrate animals
may have evolved utilizing the this combination and that we can develope a
deeper awarness of our inner structure by tracing this pattern**